

The Best Angle in Intersection Method

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ABSTRACT: *In the mapping work, the accuracy of the resulted mapping dependent on the accuracy of the framework used. Framework basic used is usually expressed in the form of control points whose coordinates are known, both temporary and permanent coordinates. The procurement and measurement of the basic framework of the mapping depend on several things, namely the purpose of measurement, area, and shape of the mapped area, the availability of equipment, ease in processing data. One method that will be used in determining the position of points at this research is intersection. A good point position can be seen based on the error ellipse shape. Determination position of the point by intersection method calculated from three points known coordinates, with a distance between each point about 100 meters and four angles located at the three points known coordinates. Input data is simulation data, the calculation is done with the matlab program version 7 and depictions with Autocad 2010. Result of the research showing the best angle at $\theta_1 = 49^\circ 8' 7''$, $\theta_2 = 60^\circ 12' 8''$, $\theta_3 = 61^\circ 21' 6''$, $\theta_4 = 80^\circ 20' 10''$. While the parameters of the long axis and short axis are $S_x = 7 \text{ mm}$, $S_y = 5 \text{ mm}$, $S_u = 5 \text{ mm}$ dan $S_v = 7 \text{ mm}$. Ellipse's error geometry is very close to the circle. The error calculation of the error ellips is*

$$Su_{95} = Su_{std}\sqrt{2F} = 0.0457$$

$$Sx_{95} = Sx_{std}\sqrt{2F} = 0.0358$$

$$Sv_{95} = Sv_{std}\sqrt{2F} = 0.0326$$

$$Sy_{95} = Sy_{std}\sqrt{2F} = 0.0431$$

KEYWORDS – *intersection, error ellipse, best angle, matlab simulation*

I. INTRODUCTION

The basic framework of the mapping is the reference point to determining the position of the detailed points that will be drawn on the map. The point that has been known or determined the horizontal position in the form of (X,Y) coordinates in a particular projection system used as a reference calculation of the coordinate of the other point. In every measurement, there must be an error and will later affect the calculation result obtained. For that, we need an appropriate adjustment computation method, to produce calculation values that are close to the actual results. In the mapping work, the accuracy of mapping produced depends on the accuracy of the basic framework used. The basic framework used is usually stated in the form of control points whose coordinates are known, both temporary and permanent coordinates. The procurement and measurement of the basic

framework of mapping depend on several things, namely:

- a. Measurement objective.
- b. The area and shape of the area being mapped.
- c. Availability of measuring instruments.
- d. Ease of data processing

The basic framework types used in the mapping field are divided into two basic, namely the basic horizontal control and the basic vertical control. The procurement of horizontal frameworks that are carried out by the terrestrial method can be done using several methods, namely:

- a. Triangulation
- b. Trilateration
- c. Series of triangles

d. Polygon

C (X_C, Y_C)

e. Resection

measured angle $\theta_1, \theta_2, \theta_3, \theta_4$

f. Intersection

Specified: P coordinates (X_P, Y_P)

Solution:

The choice of basic framework types to be used is usually adjusted to the needs in the mapping field. One of the basic horizontal control that will be discussed in this paper is the intersection method, this method is the determination of the position of a point that is calculated from three points known coordinates and measured 4 angles at points known coordinates. In some surveys, this method is often used as a basic framework for mapping. Determination of the position of the point in an intersection is very important considering that as a basic framework of mapping, detailed accuracy on the surface of the earth depicted on the map can be precisely in its position because the basic framework of this mapping is a framework that binds details. The basic framework functions in mapping, among others, as a basic horizontal control in the measurement area, as a distance and angle control, as well as a base point for further measurements so that it can facilitate the calculation and plotting of the map.

- a. Calculate azimuth AB (Ψ_{AB}), length AB (d_{AB}), azimuth BC (Ψ_{BC}), length BC (d_{BC}).

$$\Psi_{AB} = \text{arc tg } \frac{XB - XA}{YB - YA}$$

$$d_{AB} = \sqrt{(XB - XA)^2 + (YB - YA)^2}$$

$$\Psi_{BC} = \text{arc tg } \frac{XC - XB}{YC - YB}$$

$$d_{BC} = \sqrt{(XC - XB)^2 + (YC - YB)^2}$$

- b. Calculate angle α and β

$$\alpha = 180^\circ - \theta_1 - \theta_2$$

$$\beta = 180^\circ - \theta_3 - \theta_4$$

- c. Calculate azimuth AP (Ψ_{AP}), length AP (d_{AP}), azimuth BP (Ψ_{BP}), length BP (d_{BP}), azimuth CP (Ψ_{CP}), length CP (d_{CP})

$$\Psi_{AP} = \Psi_{AB} - \theta_1$$

$$\Psi_{BP} = \Psi_{AB} \pm 180^\circ + \theta_2$$

$$\Psi_{CP} = \Psi_{BC} \pm 180^\circ + \theta_4$$

$$d_{AP} = \frac{\sin \theta_2}{\sin \alpha} d_{AB}$$

$$d_{BP} = \frac{\sin \theta_1}{\sin \alpha} d_{AB}$$

$$d_{CP} = \frac{\sin \theta_3}{\sin \beta} d_{BC}$$

- d. Calculate coordinate of P

$$X_P = X_A + d_{AP} \sin \Psi_{AP}$$

II. LITERATURE REVIEW

II.1 The Theory of Intersection

The intersection method that often used in the field can be showing in the picture following:

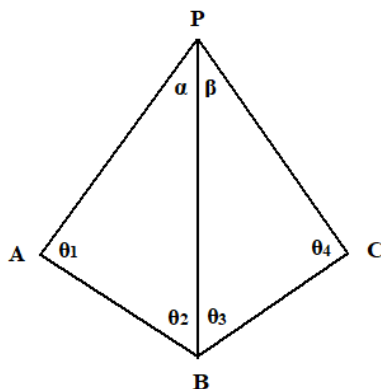


Figure 2.1 Intersection

Known: coordinates A (X_A, Y_A)

B (X_B, Y_B)

$$Y_P = Y_A + d_{AP} \cos \Psi_{AP}$$

$$X_P = X_B + d_{BP} \sin \Psi_{BP}$$

$$Y_P = Y_B + d_{BP} \cos \Psi_{BP}$$

$$X_P = X_C + d_{CP} \sin \Psi_{CP}$$

$$Y_P = Y_C + d_{CP} \cos \Psi_{CP}$$

(Wolf, 1980)

Thus the coordinates of point P can be calculated from points A, B, and C.

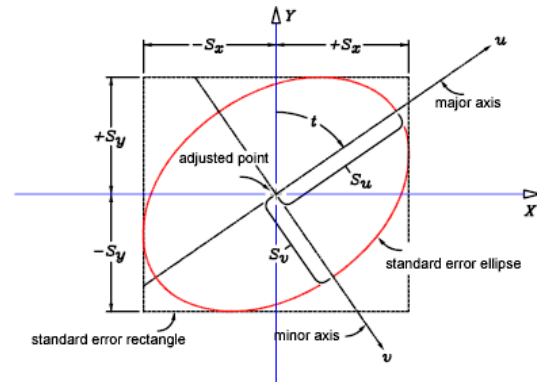
II.2 Ellipse Theory of Errors

Ellipse error is one of the parameters used to assess the quality of the relative coordinates obtained from the process of determining coordinates, both with the resection method, intersection, triangulation, trilateration, triangulation, as well as data processing in photogrammetry. The relative error ellipse is also called the error line ellipse which represents the region of confidence ("confidence region") of the accuracy of the coordinates of a point relative to other points. The shape and size of the relative error ellipse are calculated based on the covariance matrix of the coordinates of a point relative to other points. The position or location of a point can be evaluated based on the shape of the error ellipse, a good point position occurs when the price of the ellipse's axes is small and close to a circle.

II.3 Determination of Error Ellipse Parameters

The shape, size, and orientation of a relative error ellipse are generally represented by parameters along the long axis (a / S_u), half short axis (b / S_v), and the direction angle of the long axis of the error ellipse (t). These three parameters are calculated based on the

matrix and standard deviation (S_o) of the results of the adjustment computation process.



Source: Docs.autodesk.com

Ellipse Error Formula

1. Major angle parameters (t)

$$\tan 2t = \frac{2Q_{xy}}{Q_{yy} - Q_{xx}} = \frac{\sin 2t}{\cos 2t}$$

t depends on the permutation of the sign generated by $\sin 2t$ and $\cos 2t$, so the actual t value in the quadrant system can be determined according to the table below:

Symbol Algebra		Quadrant
Sin 2t	Cos 2t	
+	+	1
+	-	2
-	-	3
-	+	4

2. Half axis length parameter (Su)

$$Q_{uu} = \frac{1}{2} (Q_{yy} + Q_{xx} + K)$$

With:

$$K = [(Q_{yy} - Q_{xx})^2 + 4(Q_{xy})^2]^{1/2}$$

$$S_u = \pm S_o \sqrt{Q_{uu}}$$

3. Short half axis parameter (Sv)

$$Q_{vv} = \frac{1}{2} (Q_{yy} + Q_{xx} - K)$$

$$S_v = \pm S_o \sqrt{Q_{vv}}$$

4. Additional parameters

Figure 2.2 Ellipse Error

$$S_x = \pm S_o \sqrt{Q_{xx}}$$

$$S_y = \pm S_o \sqrt{Q_{yy}}$$

(Wolf, 1980)

II. 4 Theory of Adjustment Computation

In determining abscissa (X) and ordinate (Y) at a point P, in the following figure:

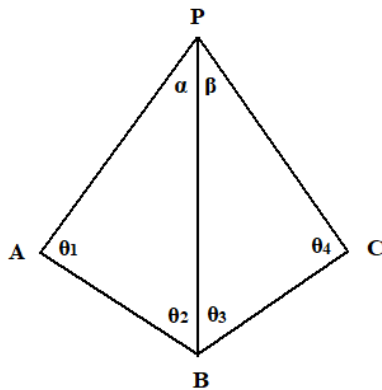


Figure 2.3. Intersection

$$X_P = X_A + d_{AP} \sin \Psi_{AP} + v_1$$

$$Y_P = Y_A + d_{AP} \cos \Psi_{AP} + v_2$$

The process of calculating the adjustment computation using the least squares method as follows:

The observation equation in the form of a matrix can be written:

$$m A_n \quad n X_1 = m L_1 + m V_1$$

$$A^T A X = A^T L$$

$$(A^T A)^{-1} (A^T A) X = (A^T A)^{-1} A^T L$$

$$I X = (A^T A)^{-1} A^T L$$

$$X = (A^T A)^{-1} A^T L$$

Standard deviation:

$$S_o = \sqrt{\frac{\sum p v^2}{n-1}}$$

To find the variance-covariance matrix the formula is:

$$S_o^2 (A^T A)^{-1}$$

With:

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ \dots & \dots \\ a_n & b_n \end{bmatrix}$$

$$X = \begin{bmatrix} X_p \\ Y_p \end{bmatrix}$$

$$L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}$$

$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

With:

A is the parameter coefficient matrix

X is a parameter

L is the price that is close to true

V is the residue

Ellips calculation error (95%) is calculated by F statistic with the formula:

$$S_{u_{95}} = S_{u_{std}} \sqrt{2F}$$

$$S_{x_{95}} = S_{x_{std}} \sqrt{2F}$$

$$S_{v_{95}} = S_{v_{std}} \sqrt{2F}$$

$$S_{y_{95}} = S_{y_{std}} \sqrt{2F}$$

The F value is read from the F table. (Wolf, 1980)

III. RESEARCH METHOD

In this study, the coordinates of the P point are calculated from three points namely A, B, and C. The known elements are the angles $\theta_1, \theta_2, \theta_3,$ and θ_4 . The calculation is done with field data by simulation the angular magnitude $\theta_1, \theta_2, \theta_3, \theta_4$ until the smallest Su, Sv, Sx and Sy prices are obtained. The calculation was performed with Matlab software version 7 and depictions performed with Autocad 2010 software.

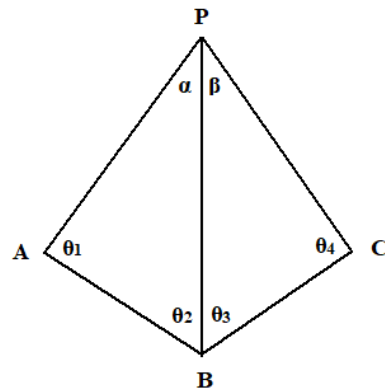
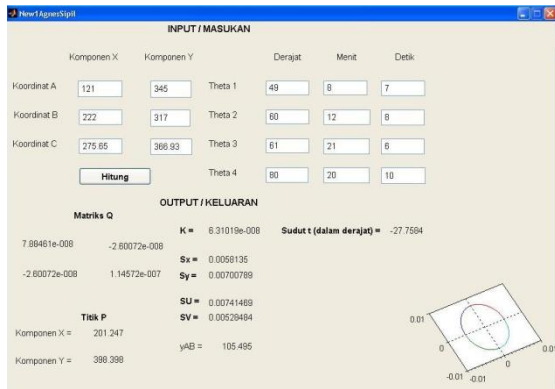


Figure 3.1 Intersection

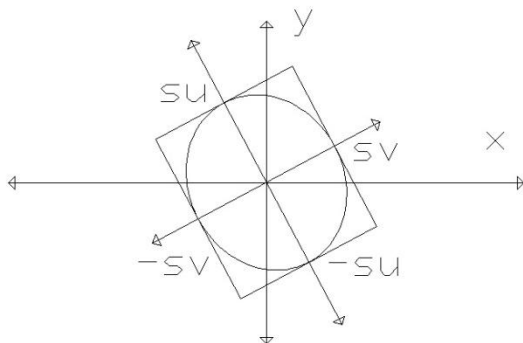
The data used in this study are AB distance of approximately 100m, BC distance of approximately 100 m and angles performed simulations, namely PAB angle (θ_1), ABP angle (θ_2), PBC angle (θ_3), and BCP angle (θ_4).

IV. RESULT AND DISCUSSION

The results of the calculation of 65 experiments can be seen in the appendix, the best angle for the angle $\theta_1, \theta_2, \theta_3, \theta_4$ can be seen below:



Whereas the error ellipse can be described as follows:

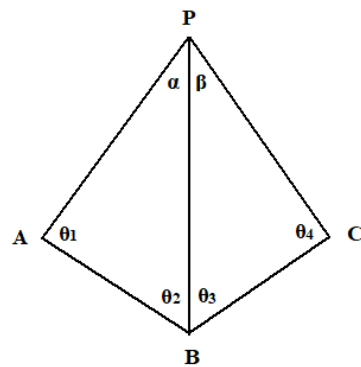


In general, the elements that are known in determining the position with the intersection method are three points with coordinates, namely point A, point B, and point C, and two angles which are also known for their magnitude, namely the APB angle and the BPC angle. In this study, in addition to three points A, B, and C, the coordinates are known, but there are 4 known angles, not APB and BPC angles, but PAB angle (θ_1), ABP angle (θ_2), PBC angle (θ_3) and BCP angle (θ_4). While the magnitude of the two APB angles and the BPC angle are calculated. The best angular results for positioning with the intersection method with a distance between two points about 100m are $\theta_1 = 49^\circ 8' 7''$, $\theta_2 = 60^\circ 12' 8''$, $\theta_3 = 61^\circ 21' 6''$, $\theta_4 = 80^\circ 20' 10''$ with the shape the

ellipse that is closest to the circle compared to other angles, namely $S_x = 7$ mm, $S_y = 5$ mm, $S_u = 5$ mm and $S_v = 7$ mm. Compared to other calculated data, this ellipse is the smallest in size. Ellipses are a good error when the prices of their axes are small and their shape approaches the circle (Waljiyanto, 1987). However, with the magnitude of the axis parameters ranging from 5 to 7 mm, it can be said that the small error ellipse and the shape close to the circle means that the position of the point P is the best.

V. CONCLUSION

Based on the results of the research conducted with simulation input data with a distance of AB and BC about 100 meters, for intersection with the following picture,



can be

concluded:

The best angle is $\theta_1 = 49^\circ 8' 7''$, $\theta_2 = 60^\circ 12' 8''$, $\theta_3 = 61^\circ 21' 6''$, $\theta_4 = 80^\circ 20' 10''$ with the ellipse shape closest to the circle compared to other angles, namely $S_x = 7$ mm, $S_y = 5$ mm, $S_u = 5$ mm and $S_v = 7$ mm with the ellipse geometry error very close to the circle.

VI. Acknowledgments

Based on the results of the research above, the author suggests that the calculation is done using length, for example, 200 meters, 300 meters, 400

meters, and the other so that it can be seen the variations of the length effect for the best angle.

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