The Mathematical Model of Long-Term Effects of Defense Expenditure on Economic Growth: a Literature Review

by Posma Sariguna Johnson Kennedy

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Abstract. Start your abstract here…

1. Introduction

According to Cornes & Sandler (1986), the activities of the government will affect the production capacity of the private sector though not reflected in prices in competitive markets. This externality, for example, can be formed from infrastructure activities, training, education, nutrition improvement, and other activities that can lead to an increase in human resources. When activities from one sector increase output from other sectors, there is positive externalities, as well as if the opposite occurs.

To see the effect of national defense on the economy over the long term, it is approached through an analysis of the supply side or production side. The effect of defense spending on economic output are through the availability of its production factors, whether labor, capital and physical capital, as well as technologies that simultaneously affect potential economic output.

One approach commonly used in research on the relationship between defense spending and economic growth is the approach of the neoclassical production function, by reviewing the supply-side description through aggregate output changes (Heo, 1998). It is also described by Ram (1986), Biswas & Ram (1986), Atesoglu and Mueller (1990), Mintz & Huang (1990, 1991), and Ward & Davis (1992). This theory is widely used because it is constructed from a consistent theory structure (Sandler & Harley, 1995).

This theory assumes that real output per capita and capital stock growth will remain at a constant level over a period of time despite short-term fluctuations. It is also assumed that an increase in labor and capital input at the steady level will also increase the aggregate output at a steady level (Solow, 1970). Then the change from the aggregate output will be explained through changes in capital and labor.

Neoclassical growth theory (Solow, 1956), explains that is no channel of government spending that affects long-term economic growth. The simplest model in shaping the economic model is to form the assumption of a closed economy and a production sector, where output is a homogeneous good that can be consumed at once invested (Barro-Martin, 1995).

Investment produces new physical capital, and the capital depreciates at a constant rate. Households and firms are considered joint units (which have input and manage technologies that transform inputs to output), the market is neglected. The output current produced at time t (Y) is affected by the production function of capital (K) and labor (L) depending on the time (t) to reflect the effects of technological development, described as follows: $Y(t) = F[K(t), L(t), t]$

However, many assumptions are developed in modeling to include government expenditures with the approach of the neoclassical production function. Government defense expenditure and its overall impact on growth can be analyzed in the context of dividing the economy into several sectors, not just one sector. These sectors form externalities that affect other sectors. The assumptions that divide macroeconomic into two sectors, government and non-government (private) are preceded by Ram (1986), Biswas and Ram (1986), based on the idea of non-export-export model by Feder (1982). Mintz and Stevenson (1995) added that the defense sector should be separated from the non-defense sector because it has different incentives.

Cornes and Sandler (1986) supported by Mintz and Huang (1990, 1991) state that government activities will affect the production capacity of the private sector without price and competition market. This creates an externality effect on the output of the private sector. Mintz and Huang decomposed government activities into military and non-military spending (Mintz and Huang 1990, 1991; Huang and Mintz 1990). By splitting aggregate government spending into military expenditures and non-military expenditures, equality is formed in three components, namely private sector production, non-military government spending, and military spending.

Subsequent model developments were proposed by Mueller and Atesoglu (1993), which incorporated technological development factors into the model, in accordance with the Solow approach (1957). According to Solow (1957), the technology is explained by how much output can be generated from labor and capital used in production. As technological developments contribute significantly to economic development, it is important to study the effect of defense spending on economic growth by incorporating technological developments in the defense-growth model. Incorporating technological developments in models is significant as technology has been reflected as part of economic and social integration (Hofheinz and Calder, 1982). Denison (1985) also explains that advanced technology will provide a way for production at a more cost-effective cost. Technological changes may occur differently on a yearly basis, but Chan (1987) assumes that in the long run develops with an average amount of e□t. By entering this rate in the model, we can see the relationship between defense spending and economic growth in non-linear form

2. The Two-Sector Growth and Defense Theory of Feder-Ram

As noted earlier, one common approach used to look at the effect of defense spending and economic growth is the approach of the neoclassical production function. That is by reviewing the supply-side description through the aggregate output changes (Uk Heo, 1998).

Antonakis (1999) argues that, it is important to build models by dividing the economy into sectors aimed at capturing the impact of military spending on growth, although not reviewing the influence of other macro variables. Thus the overall influence of military spending on growth is analyzed in the context of dividing the economy into several sectors, in which sectors form externalities that affect other sectors.

The first two-sector economic model established to look at the effects of defense spending and the economy is a model of Ram (1986) based on the neoclassical production function approach built by Feder (1983) and Deninson's (1985). Feder (1983) writes that aggregate growth is related to changes in capital and labor through a certain production function (underlying production function). He built a two-sector production function model consisting of export and non-export sectors. Deninson's source-of-growth model uses a supply-side description of aggregate output change, which explains aggregate growth in its changes to capital and labor.

Based on their views Feder (1983) and Deninson's (1985), Ram (1986) built a model of two sectors also by comprising the government sector and the private sector. In the development of defense economics, the Feder-Ram model is widely used in explaining the relationship between military budget and economic growth from the supply side.

The two-sector model built by Ram (1986) is the military output sector (M, government) and civil output sector (C, private sector). Both sectors use labor (L) and capital (K), while the production function of the military will exert external effects on production from the private sector. The aggregate production function of the output of the economic sectors is given by the following functions:

$$M = M(L_m, K_m)$$
$$C = C(L_c, K_c) = M^{\theta} c(L_c, K_c) = C(L_c, K_c, M) \tag{1}$$

Limitations of endowment factors are as follows:

$$L = \sum_{i \in s} L_i$$
$$K = \sum_{i \in s} K_i \qquad S = \{m, c\} \tag{2}$$

And the national income is:

$$Y = C + M \tag{3}$$

The sum of these "butter" and "guns" can only be understood if their value is monetary output compared to the quantity of output. It would therefore be better to be formed in the normalization of prices, as follows:

$$Y = P_c.Cr.(L_c, K_c) + P_m.Mr.(L_m, K_m) \tag{3a}$$

where Pm and Pc are constant unitary units of money prices associated with the quantity of output of *Mr.* and *Cr*. From this model can be formed marginal products of both labor (M_L, C_L) and capital (M_K, C_K) which can be made proportionally between sectors, is a derivative of the production function of inputs, namely:

$$\frac{M_L}{C_L} = \frac{M_K}{C_K} = 1 + \delta \tag{4}$$

The notations in equation (4), C_M show the external effects of military output to the private sector, and δ denote the relative factor productivity difference between the two sectors. If $C_M > 0$ and/ or $\delta > 0$, an increase in military output will cause a large growth rate of total Y output, from the sum of M and C, using the given production function L and K.

Equation (4) is equivalent to:

$$\frac{P_m.Mr_L}{P_c.Cr_L} = \frac{P_m.Mr_K}{P_c.Cr_K} = 1 + \delta \tag{4a}$$

Equation (4a) shows the comparison between the productivity of marginal factors between different production depending on the prices used on sectoral outputs.

Differentiation of (3), (1), and (2) forms the econometric speech of the growth equation, as follows:

$$\dot{Y} = \frac{C_L L}{Y}\dot{L} + C_K \cdot \frac{I}{Y} + \left(\frac{\delta}{1+\delta} + C_M \right)\frac{M}{Y}\dot{M} \tag{5}$$

where the dot notation indicates the rate of change in proportion or growth rate. I/Y and M/Y are notations of the ratio of investment and military spending to total output. While $I = dK$ which is a net investment (net investment).

Using equation (1) and the constant elasticity of C to M equation (5) can be changed in the form of:

$$\dot{Y} = \alpha\left(\frac{I}{Y}\right) + \beta(\dot{L}) + \left(\frac{\delta}{1+\delta} - \theta \right)\frac{M}{Y}\dot{M} + \theta.\dot{M} \tag{6}$$

Where :

$$\dot{Y} = \left[\frac{dY}{Y}\right] \qquad \dot{L} = \left[\frac{dL}{L}\right] \qquad \dot{M} = \left[\frac{dM}{M}\right]$$

$$\theta = C_M\left[\frac{M}{Y-M}\right] \qquad \alpha = C_K \qquad \beta = \left[\frac{C_L.L}{Y}\right]$$

The difference in (5) with (6) is that in equation (5) hypothesis testing is possible only if the variables C_M and δ are zero. However this would in fact cause the coefficient of (M/Y) to be zero thus reducing the estimate of the growth equation (standard growth equation). If using equation (6) can be estimated separately to identify separately from the "externality effect" C_M and the "marginal factor productivity differential effect" δ.

3. Three-Sector Growth and Defense Theory of Mintz-Huang

The next model of Feder-Ram was developed by Huang & Mintz (1990), Mintz & Huang (1990, 1991) with some modifications. Mintz & Huang (1990, 1991), argued that the externality effect of the military and non-military sectors is different. Therefore, the effects of externalities of the military public sector, the non-military public sector, and the private sector are included in the production function separately. The review of the effects of these externalities is done because their influence is not reflected in market prices (Cornes & Sandler, 1986).

Mueller and Atesoglu (1993) incorporated technological development factors into the model, in accordance with the Solow (1957) approach. Solow said the technology is explained by how much output can be generated from labor and capital used in production. As technological developments contribute significantly to economic development, it is important to study the effect of defense spending on economic growth by incorporating technological developments in the defense-growth model. Incorporating technological developments in models is significant as technology has been reflected as part of economic and social integration (Hofheinz and Calder, 1982). Denison (1985) also explains that advanced technology will provide a way for production at a more cost-effective cost. Technological changes may occur differently on a yearly basis, but Chan (1987) assumes that in the long run develops with an average amount of $e^{\lambda t}$. By entering this rate in the model, we can see the relationship between defense spending and economic growth in non-linear form.

Gross domestic income (economic output) is the sum of accounting of expenditures from individual consumption, capital or investment formation, government spending on goods and services, and net foreign trade in the form of total exports minus total imports. Each category can be further broken down. Model improvements and modifications continue to be made by various economists.

By splitting aggregate G into military expenditure (M) and non-military expenditure (N), Mintz & Huang (1990) form the equation into three components, namely private sector (P) production, non-military government expenditure (N), and military expenditure (M), so that:

$$Y = C + I + G + (EX - IM) = [C + I + (EX - IM)] + N + M$$

$$So \;\; Y = P + N + M \qquad \text{(The Mintz-Huang Model)} \tag{7}$$

According to Mintz & Huang (1990, 1991) argue that the effect of externalities of the government spending sectors on military and non-military varies, therefore it has different production functions. The production function model becomes:

$$M = A(t)F(L_m, K_m);\; N = B(t)G(L_n, K_n);\; P = C(t)H(L_p, K_p, M, N) \tag{L.22}$$

Level of technological development between sector-based sectors P (private) is written as follows:

$$A(t)/C(t) = 1 + \Phi_m;\; B(t)/C(t) = 1 + \Phi_n; \tag{L.23}$$

The marginal productivity of labor and capital can be written on the basis of P (private) sector as follows:

$$F_l/H_l = F_k/H_k = 1 + \delta_m;\; G_l/H_l = G_k/H_k = 1 + \delta_n \tag{9}$$

Where the total input is:

$$L = L_m + L_n + L_c;\; K = K_m + K_n + K_c. \tag{9}$$

The economy grows over time, from equation (1) can be differentiation with respect to time of each equation, that are:

waktu t (*differentiation with respect to time*)[1] dari setiap persamaan (*equation*), yaitu :

$$M = A(t)F(L_m, K_m) \qquad \text{so } M' = dM = F.dA + A.F_l dL_m + A.F_k dK_m$$

$$N = B(t)G(L_n, K_n) \qquad \text{so } N' = dN = G.dB + B.G_l dL_n + B.G_k dK_n$$

$$P = C(t)H(L_p, K_p, M, N) \qquad \text{so } P' = dP = HdC + CH_l dL_p + CH_k dK_p + CH_m dM + CH_n dN \tag{L.28}$$

Total differential sum of all outputs gives result:

$$dY = FdA + AF_l dL_m + AF_k dK_m + GdB + BG_l dL_n + BG_k dK_n + HdC + CH_l dL_p + CH_k dK_p + CH_m dM + CH_n dN \tag{L.29}$$

Using the marginal productivity equation and collecting the same terms, it is found:

$$dY = [FdA + GdB + HdC] + [H_l(AdL_m + BdL_n) + H_k(AdK_m + BdK_n)] + [\delta_m A(H_l dL_m + H_k dK_m) + \delta_n B(H_l dL_n + H_k dK_n)] + (CH_m dM + CH_n dN) \tag{L.30}$$

Using the technological development equation:

$$A(t)/C(t) = 1 + \Phi_m; \quad C(t) = A(t)/(1 + \Phi_m); \quad A(t) = C(t).(1 + \Phi_m)$$

$$B(t)/C(t) = 1 + \Phi_n; \quad C(t) = B(t)/(1 + \Phi_n); \quad B(t) = C(t).(1 + \Phi_n) \tag{L.31}$$

and from the differentiation of time t then:

$$dL = dL_m + dL_n + dL_p \quad dan \quad dK = dK_m + dK_n + dK_p$$

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[1] *Misalnya, X'(t)=dX(t)/dt yang kemudian disingkat menjadi dX*

(L.32)

Collecting the same variables gives:

$$dY = (FdA + GdB + HdC) + (CH_l dL + CH_k dK) + (\Phi_m CH_k dK_m + \Phi_n CH_k dK_n)] + [(\Phi_m CH_l dL_m + \Phi_n CH_l dL_n) + [\delta_m A(H_l dL_m + H_k dK_m) + \delta_n B(H_l dL_n + H_k dK_n)] + (CH_m dM + CH_n dN)$$

(L.33)

Can be simplified to:

$$dY = (FdA + GdB + HdC) + (CH_l dL + CH_k dK) + \frac{\Phi_m}{1+\Phi_m} A.H_l.dL_m + \delta_m A.H_l.dL_m + \frac{\Phi_m}{1+\Phi_m} A.H_k.dK_m + \delta_m A.H_k.dK_m + \frac{\Phi_n}{1+\Phi_n} B.H_l.dL_n + \delta_n B.H_l.dL_n + \frac{\Phi_n}{1+\Phi_n} B.H_k.dK_n + \delta_n B.H_k.dK_n + (CH_m dM + CH_n dN)$$

(L.34)

Using the marginal productivity equation, obtained:

$$dY = (FdA + GdB + HdC) + (CH_l dL + CH_k dK) + \frac{\Phi_m + \delta_m + \Phi_m\delta_m}{(1+\Phi_m)(1+\delta_m)} A.F_l.dL_m + \frac{\Phi_m + \delta_m + \Phi_m\delta_m}{(1+\Phi_m)(1+\delta_m)} A.F_k.dK_m + \frac{\Phi_n + \delta_n + \Phi_n\delta_n}{(1+\Phi_n)(1+\delta_n)} B.G_l.dL_n + \frac{\Phi_n + \delta_n + \Phi_n\delta_n}{(1+\Phi_n)(1+\delta_n)} BG_k dK_n + (CH_m dM + CH_n dN)$$

(L.35)

By collecting the same terms, the equation becomes:

$$dY = (FdA + GdB + HdC) + (CH_l dL + CH_k dK) + \frac{\Phi_m + \delta_m + \Phi_m\delta_m}{(1+\Phi_m)(1+\delta_m)} (A.F_l.dL_m + A.F_k.dK_m) + \frac{\Phi_n + \delta_n + \Phi_n\delta_n}{(1+\Phi_n)(1+\delta_n)} (B.G_l.dL_n + BG_k dK_n) + (CH_m dM + CH_n dN)$$

(L.36)

so:

$$dY = CH_l dL + CH_k dK + \left(\frac{\Phi_m + \delta_m + \Phi_m\delta_m}{1+\delta_m}\right)\left(\frac{1}{1+\phi_m}\right)(AF_l dL_m + AF_k dK_n + FdA) + \left(\frac{\Phi_n + \delta_n + \Phi_n\delta_n}{1+\delta_n}\right)\left(\frac{1}{1+\phi_n}\right)(BG_l dL_n + BG_k dK_n + GdB) + \left(1 - \frac{\Phi_m + \delta_m + \Phi_m\delta_m}{(1+\delta_m)(1+\phi_m)}\right)FdA + \left(1 - \frac{\Phi_n + \delta_n + \Phi_n\delta_n}{(1+\delta_n)(1+\Phi_n)}\right)GdB + HdC + CH_m dM + CH_n dN$$

(L.37)

By:

$$dM = F.dA + A.F_l.dL_m + A.F_k.dK_m; \quad dN = G.dB + B.G_l.dL_n + B.G_k.dK_n$$

(L.38)

equations can be simplified:

$$dY = CH_l dL + CH_k dK + \left(\frac{\phi_m + \delta_m + \phi_m\delta_m}{(1+\delta_m)(1+\phi_m)} + CH_m\right)dM + \left(\frac{\Phi_n + \delta n_n + \Phi_n \delta n_n}{(1+\delta_n)(1+\Phi_n)} + CH_n\right)dN + \left(1 - \frac{\Phi_m + \delta_m + \Phi_m\delta_m}{(1+\delta_m)(1+\Phi_m)}\right)FdA + \left(1 - \frac{\Phi_n + \delta n_n + \Phi_n \delta n_n}{(1+\delta_n)(1+\Phi_n)}\right)GdB + HdC$$

(L.39)

If, π_i is a constant and involving parameter Φ_i, δ_i,

$$\pi_i = \frac{(\Phi_i + \delta_i + \Phi_i\delta_i)}{(1+\Phi_i)(1+\delta_i)} \qquad (i = m, n,)$$

(L.40)

Where δ_i is a factor productivity differential of unknown constants that can be of any value, including zero, and Φ_i is a factor of the proportion of technological developments, of unknown constants.

Then:

$$dY = CH_l dL + CH_k dK + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + (1-\pi_m)FdA + (1-\pi_n)GdB + HdC \tag{L.41}$$

so :

$$dY = CH_l dL + CH_k dK + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + (1-\pi_m)FdA + (1-\pi_n)GdB + HdC \tag{L.42}$$

Using the equation:

$$\begin{aligned} &A(t)/C(t) = 1+\Phi_m\ ; \qquad A(t) = C(t).(1+\Phi_m) \\ &dA = dC + \Phi_m.dC = dC\ (1+\Phi_m) \\ &B(t)/C(t) = 1+\Phi_n\ ; \qquad B(t) = C(t).(1+\Phi_n) \\ &dB = dC + \Phi_n.dC = dC\ (1+\Phi_n) \end{aligned} \tag{L.43}$$

then:

$$dY = CH_l dL + CH_k dK + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + (1-\pi_m)F(1+\Phi_m)dC + (1-\pi_n)G(1+\Phi_n)dC + HdC \tag{L.44}$$

If the constants:

$$\varpi_s = [(1-\pi_m)(1+\Phi_m)(1+\delta_m)] = \frac{(1+\Phi_m+\delta_m+\Phi_m\delta_m)-(\Phi_m+\delta_m+\Phi_m\delta_m)}{(1+\Phi_m)(1+\delta_m)}.(1+\Phi_m)(1+\delta_m) = 1 \tag{L.45}$$

then :

$$dY = CH_l dL + CH_k dK + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + dC.[(1-\pi_m)(1+\Phi_m)F + (1-\pi_n)(1+\Phi_n)G + H] \tag{L.46}$$

If :

$$1-\pi_i = 1 - \frac{(\Phi_i+\delta_i+\Phi_i\delta_i)}{(1+\Phi_i)(1+\delta_i)} = \frac{1}{(1+\Phi_i)(1+\delta_i)} \tag{L.47}$$

then :

$$dY = CH_l dL + CH_k dK + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + dC\left[\left(\frac{1}{1+\delta_m}\right)F + \left(\frac{1}{1+\delta_n}\right)G + H\right] \tag{L.48}$$

By dividing each side of the equation by the total output, Y, it can be written in terms of growth rates:

$$\frac{dY}{Y} = CH_l\left(\frac{dL}{L}\right)\left(\frac{L}{Y}\right) + CH_k\left(\frac{dK}{K}\right)\left(\frac{K}{Y}\right) + (\pi_m + CH_m)dM + (\pi_n + CH_n)dN + \frac{dC\left[\left(\frac{1}{1+\delta_m}\right)F + \left(\frac{1}{1+\delta_n}\right)G + H\right]}{C[(1+\Phi_m)F + (1+\Phi_n)G + H]} \tag{L.49}$$

where:

$$Y = M + N + P = AF + BG + CH = C[(1+\Phi_m)F + (1+\Phi_n)G + H] \tag{L.50}$$

Assuming the rate of technological development as a constant exponential level, by declaring the private sector as:

$$C = e^{\lambda t} \tag{L.51}$$

Then substituting it with the previous equation gives:

$$\frac{dY}{Y} = e^{\lambda t} H_l \left(\frac{dL}{L}\right)\left(\frac{L}{Y}\right) + e^{\lambda t} H_k \left(\frac{dK}{K}\right)\left(\frac{K}{Y}\right) + \left(\pi_m + e^{\lambda t} H_m\right)\left(\frac{dM}{M}\right)\left(\frac{M}{Y}\right)$$
$$+ \left(\pi_n + e^{\lambda t} H_n\right)\left(\frac{dN}{N}\right)\left(\frac{N}{Y}\right) + \frac{\lambda.\left[\left(\frac{1}{1+\delta_m}\right)F + \left(\frac{1}{1+\delta_n}\right)G + H\right]}{\left[(1+\Phi_m)F + (1+\Phi_n)G + H\right]} \tag{L.52}$$

By using:

$$F = M/A = M/[C(1+\phi_m)];\ G = N/B = N/[C(1+\phi_n)];\ H = P/Y \text{ dan } P = Y - M - N \tag{L.53}$$

the equation becomes:

$$\frac{dY}{Y} = e^{\lambda t} H_l \left(\frac{dL}{L}\right)\left(\frac{L}{Y}\right) + e^{\lambda t} H_k \left(\frac{dK}{K}\right)\left(\frac{K}{Y}\right) + \left(\pi_m + e^{\lambda t} H_m\right)\left(\frac{dM}{M}\right)\left(\frac{M}{Y}\right) + \left(\pi_n + e^{\lambda t} H_n\right)\left(\frac{dN}{N}\right)\left(\frac{N}{Y}\right)$$
$$+ \frac{\lambda.\left[\left(\frac{1}{1+\delta_m}\right)\left(\frac{M}{(1+\Phi_m)C}\right) + \left(\frac{1}{1+\delta_n}\right)\left(\frac{N}{(1+\Phi_n)C}\right) + \frac{P}{C}\right]}{\left[(1+\Phi_m)\left(\frac{M}{(1+\Phi_m)C}\right) + (1+\Phi_n)\left(\frac{N}{(1+\Phi_n)C}\right)G + \left(\frac{Y-M-N}{C}\right)\right]} \tag{L.54}$$

Thus the equation can be simplified as follows:

$$\frac{dY}{Y} = \lambda + e^{\lambda t} H_l \left(\frac{L}{Y}\right)\left(\frac{dL}{L}\right) + e^{\lambda t} H_k \left(\frac{K}{Y}\right)\left(\frac{dK}{K}\right) + [\pi_m + e^{\lambda t} H_m]\left(\frac{dM}{M}\right)\left(\frac{M}{Y}\right)$$
$$+ [\pi_n + e^{\lambda t} H_n]\left(\frac{dN}{N}\right)\left(\frac{N}{Y}\right) + \lambda\pi_m\left(\frac{M}{Y}\right) + \lambda\pi_n\left(\frac{N}{Y}\right) \tag{L.55}$$

By ψ_i is the externality of sector-*i* to private sector:

$$\psi_l = H_l.(L/Y)\ ;\ \psi_k = H_k.(dK/K)\ ;\ \psi_s = H_s.(S/Y)\ ;\ \psi_m = H_m.\omega.(M/Y)\ ;\ \text{dan}\ \ \psi_n = H_n.\omega.(N/Y) \tag{L.56}$$

Equations can be changed in order to be able to estimate, namely:

$$\frac{dY}{Y} = \lambda + e^{\lambda t}\psi_l\left(\frac{dL}{L}\right) + e^{\lambda t}\psi_k\left(\frac{I}{Y}\right) + [\pi_m(M/Y) + e^{\lambda t}\psi_m]\left(\frac{dM}{M}\right) + [\pi_n(N/Y) + e^{\lambda t}\psi_n]\left(\frac{dN}{N}\right) + \lambda\pi_m\left(\frac{M}{Y}\right) + \lambda\pi_n\left(\frac{N}{Y}\right) + \varepsilon_t \tag{L.57}$$

4. Criticism of the Growth-Defense Model

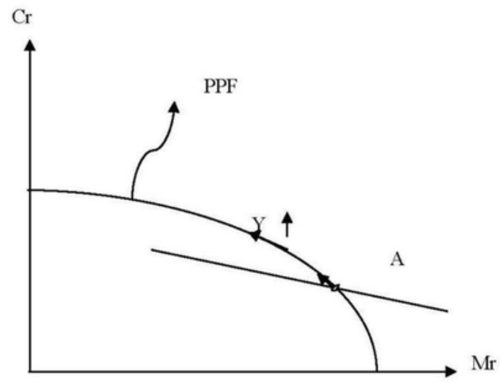
Much debate has emerged against the defense-growth model that begins with the Feder-Ram model. Some experts provide support, such as Deger and Sen (1995) that characterize the externality model of Feder-Biswas-Ram as "a splendid empirical workhorse to investigate the impact of military expenditure on growth." Mintz and Stevenson (1995) seen using formal justification to include military expenditure as explanatory variables in growth regression analysis with single-equation growth analysis, based on neoclassical growth theory as its foundation. Or, Biswas and Ram (1986) state, at least stands out well on the framework of neoclassical production functions. His famous approach is the emergence of a direct link from the theoretical model to the econometric specification.

However, many critics say that Ram's study does not include other independent variables that also affect economic growth. From the Keynesians see that neoclassical approach is not able to answer the problem in the short term especially if there are shock-shock encountered. Full economic conditions are only limited to long-term models that are difficult to implement in reality. Even in the formation of his model faced many inaccuracies, as described by Dunne, Smith, and Willenbockel (2004) below.

Feder-Ram model that notation of differences in factor of marginal productivity between sectors, as in the following:

$$\frac{M_L}{C_L} = \frac{M_K}{C_K} = 1 + \delta \tag{28}$$

actually gives some wrong interpretations. In empirical literature, non-zero is generally interpreted to reflect a situation in which one sector is more efficient or less prod- uctive in its factor users than others.



For example, Ward et al (1993) estimates a negative sign for Taiwan which can be inferred "that in comparison to the civil sector ..., the military sector is realized more inefficiently". Antonakis (1997) and Atesoglu and Mueller (1990) write something similar: "Without a strong competitive pressure that leads to ... efficiency in the management and use of resources, it can be argued that marginal productivity factors are much lower in the defense sector".

Such interpretations do not conform to a recognized model of theory. Although this seems to have never been observed in the literature, the technical efficiency in production held in the assumption model: with the use of similarity to factor productivity differentials for both factors, civil and military through the previous equation, a study based on two-sector model Feder -Ram actually assumes that the economy produces at the border of the production possibility frontier (*PPF*) set, as shown below:

In this context, technical efficiency in production is achieved when production C can not increase without compromising the production of M, or vice versa. This shows the equalization between marginal rates of technical substitution (*MRTS*) between labor and capital across the production sectors. When $MRTS_M = M_{rK}/M_{rL}$ and $MRTS_C = C_{rK}/C_{rL}$, efficiency conditions can be restated in the form of $M_{rK}/M_{rL} = C_{rK}/C_{rL}$, the same as the Feder-Ram model shaped:

$$\frac{P_m Mr_L}{P_c Cr_L} = \frac{P_m Mr_K}{P_c Cr_K} = 1 + \delta \tag{29}$$

This shows that the non-zero describes a number of specific sectors of inefficiency in the use of its resources is a defective form. the non-zero occurs when the implicit price ratio $P = Pm / Pc$ is used in evaluating real GDP deviating from the marginal rate of transformation (*MRT*) between *Cr* and *Mr*, which measures how many "butter" are sacrificed in producing other "gun".

When $P<MRT$, $\delta < 0$, and real GDP is calculated according to the Feder-Ram model according to the equation:

$$Y = P_c.Cr.(L_c, K_c) + P_m.Mr.(L_m, K_m) \tag{30}$$

will also increase if resources move from military production to civilian, or vice versa if $P>MRT$ and $\delta > 0$. However, GDP growth through factor reallocation is not an excuse for shifting resources from sector to inter-sectoral resource management inefficiently due lacking competitive pressure on the *less organizational slack* sector.

In the case of the above figure, real GDP increases by moving the resources from M to C, since at point A the value of a Cr unit in the form of Mr ( ) goods in the form of calculations from Y is higher than the social cost in meproduksi other units of Cr in the form of Mr (I/MRT).

Another potential argument that the approach that is supposed to capture some of the behavior of off-the-production functions is not appropriate. The production function of the Feder-Ram model:

$$\begin{aligned} M &= M(L_m, K_m) \\ C &= C(L_c, K_c) = M^{\theta} c(L_c, K_c) = C(L_c, K_c, M) \end{aligned} \tag{31}$$

used for the derivation of the empirical growth equation:

$$\dot{Y} = \frac{C_L.L}{Y}\dot{L} + C_K.\frac{I}{Y} + \left(\frac{\delta}{1+\delta} + C_M \right)\frac{M}{Y}\dot{M} \tag{32}$$

Or

$$\dot{Y} = \frac{C_L.L}{Y}\dot{L} + C_K.\frac{I}{Y} + \left(\frac{\delta}{1+\delta} - \theta \right)\frac{M}{Y}\dot{M} + \theta\dot{M} \tag{33}$$

is specific to a certain level which is not different for intra-sectoral organizations. The model was built incapable to calculate the intra-oriental organizational inefficiency. The deeper question is whether the migration of a resource that raises real GDP is what the social really wants cannot be answered without knowing where the relative P price is used in an adequate Y calculation that reflects the social marginal rate of substitution, the exchange of M for C. If that is the case, the non-zero δ reflects the situation of the product mix in a large economy and the allocation of inter-sectoral factors in eeconomy as a whole is inefficient, there is not much we can do to convert inputs into outputs in individual sectors. In theoretically, there are many econometric problems in estimating the Feder-Ram model. In the Feder-Ram equation where econometrically it can be derived as:

$$\dot{Y} = \beta_1\dot{L} + \beta_2\left(\frac{I}{Y}\right) + \beta_3\frac{M}{Y}\dot{M} + \beta_4\dot{M} + \varepsilon \tag{34}$$

It can not be indicated which is a variable and which serves as a parameter. This equation treats capital (as a form of influence) and is asymmetric, and it is clear that $C_L L/Y$ chills as a constant β_1, $C_K I/Y$ compilation is divided as a parameter and a variable, $\beta_2 . I/Y$. It is not clear from where error origin and why it is treated as white noise.

4. Conclusion

Nothing else comes from military externalities, although this is specifically used as an intercept in the above equation. As for the various simultaneous problems by creating a growth rate of the same dimensions, when the combination of the military is constant, the net output will determine the movement of the military. The multicolinearity between the two final forms yields a large standard error and the estimated inaccuracy (not appropriate forecast) of the parameters of the externalities. The model is static, there is no lagged regressors or dependent variables, which pose a major problem in time-series in cross-section.

Wijeweera & Webb (2009) examined the relationship between growth and economic growth for Sri Lanka using the Feder-Ram usage model (Wilkins, 2004) and Keesian Models of Atesoglu (2002) and Halicioglu (2004). In the Feder-Ram model the results showed t-statistics for each major coefficient (*L* and *K*) were positive, but not statistically significant in the experiments included for military-controlled operations (*M*). Since wages are insignificant it can not be censored whether or not pngelpangin positively affects economic growth in Sri Lanka. Not only are the coefficients sttistically insignificant, but R2 is also very low which means that the model is not strong enough to get out of the economy. All this leaves doubt on the Feder-Ram model to see the relationship between space and economic growth in the case of Si Lanka. [Wijeweera & Webb, 2009]

If the flaws here and there, the Feder-Ram model is very broad by people where its limitations are tried to be overcome by various circles. As with the development of a three-sector growth-defense model from Mintz-Huang (1990, 1991) which highlights on the sectors of economics and its externalities, Mueller & Atesoglu (1993) examined technology (technological change), sectoral growth-defense model with technology from UK Heo and & DeRouen (1998), as well as a four-sector growth-defense model (Antonakis 1999, Cuaresma & Reitschuler, 2004, and UK Heo & Eger, 2005) that create non-linear effects on military spaces and other sectors. Dunne, Smith, and Willenbockel (2004) continue to criticize by stating that there is no solid basis and econometric reason for using this Feder-Ram model.

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