Mathematical Model: The Long-Term Effects of Defense Expenditure on Economic Growth and the Criticism

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Mathematical Model: The Long-Term Effects of Defense Expenditure on Economic Growth and the Criticism

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Abstract. This paper aims to form a mathematical model that can explain the influence of national defense budget on the economy through economic growth. The research method is by literature review from the work of economists in making the production function model that can bring defense expenditure into the economic growth model. The model is two-sector economic model, established to look at the effects of defense spending and the economy. The model of Ram based on the neoclassical production function approach. The externality effect of the military and non-military sectors is different, so, they are included in the production function separately.

1. Introduction

According to Cornes & Sandler, the activities of the government will affect the production capacity of the private sector though not reflected in prices of competitive markets. This externality, for example, can be formed from infrastructure activities, training, education, nutrition improvement, and other activities that can increase the human resources an. When activities of one sector can increase output from other sectors, there is a positive externalities, as well as if the opposite occurs. [1]

To see the effect of national defense on the economy over in the long term, it is approached through an analysis of the supply side or production side. The effect of defense spending on economic output are through the availability of its production factors, whether labor, capital and physical capital, as well as technologies that simultaneously affect potential economic output.

Based on a formal model framework, a mathematical model would like to look at the effect of defense spending on economic growth. Although in reality the relationship is very complex, but we make a mathematical simplification through economic growth model. In general, this study aims to form a mathematical model that can explain the influence of national defense budget on the economy through economic growth. The research method is by literature review from the work of economists in making the production function model that can bring defense expenditure into the economic growth model.

One approach that commonly used in research on the relationship between defense spending and economic growth is the neoclassical production function approach, by reviewing the supply-side description through aggregate output changes [2]. It is also described by Ram [3], Biswas & Ram [4],

Atesoglu and Mueller [5], Mintz & Huang [6][7], and Ward & Davis [8] and Heo [2]. This theory is widely used because it is constructed from a consistent theory structure.[9]

This theory assumes that real output per capita and capital stock growth will remain in a constant level over a period of time despite short-term fluctuations. It is also assumed that an increase in labor and capital input at the steady level will also increase the aggregate output at a steady level. [10] Then the change from the aggregate output will be explained through changes in capital and labor.

In neoclassical growth theory, explains that is no channel of government spending that affects long-term economic growth.[11] The simplest model in shaping the economic model is to form the assumption of a closed economy and a production sector, where output is a homogeneous good that can be consumed at once invested. Investment produces new physical capital, and the capital depreciates at a constant rate. Households and firms are considered joint units (which have input and manage technologies that transform inputs to output), the market is neglected. The current output produced at time t (*Y*) is affected by the production function of capital (*K*) and labor (*L*) depending on the time (t) to reflect the effects of technological development, described as follows *Y (t) = F [K (t), L (t), t]*. [12]

However, many assumptions are developed in modeling to include government expenditures with the approach of the neoclassical production function. Government defense expenditure and its overall impact on growth can be analyzed in the context of dividing the economy into several sectors, not just one sector. These sectors form externalities that affect other sectors. The assumptions that divide macroeconomic into two sectors, government and non-government (private) are preceded by Ram [3], Biswas and Ram [4], based on the idea of non-export-export model by Feder [13].

2. Method

One common approach used to look at the effect of defense spending and economic growth is the neoclassical production function approach. That is by reviewing the supply-side description through the aggregate output changes.[2][14]

Antonakis argues that, it is important to build models by dividing the economy into sectors aimed at capturing the impact of military spending on growth, although not reviewing the influence of other macro variables. Thus the overall influence of military spending on growth is analyzed in the context of dividing the economy into several sectors, in which sectors form externalities that affect other sectors.[15]

The two-sector economic model established to look at the effects of defense spending and the economy is a model of Ram based on the neoclassical production function approach built by Feder and Deninsons. [3] Feder writes that aggregate growth is related to changes in capital and labor through a certain or underlying production function.[13] He built a two-sector production function model, consisting of export and non-export sectors. Deninson's source-of-growth model uses a supply-side description of aggregate output change, which explains aggregate growth in its changes to capital and labor.[16] Based on their views Feder, Deninson, and Ram built a model of two sectors also by comprising the government sector and the private sector. In the development of defense economics, the Feder-Ram model is widely used in explaining the relationship between military budget and economic growth from the supply side.

The two-sector model built by Ram is the military output sector (*M*, government) and civil output sector (*C*, private sector). Both sectors use labor (*L*) and capital (*K*), while the production function of the military will exert external effects on production from the private sector. [3] The aggregate production function of the output of the economic sectors is given by the following functions:

$$M = M(L_m, K_m)$$
$$C = C(L_c, K_c) = M^{\theta} c(L_c, K_c) = C(L_c, K_c, M) \quad (1)$$

Limitations of endowment factors are as follows:

$$L = \sum_{i \in s} L_i \qquad K = \sum_{i \in s} K_i \qquad S = \{m, c\} \tag{2}$$

And the national income is:

$$Y = C + M \tag{3}$$

The sum of these "butter" and "guns" can only be understood if their value is in monetary output compared to the quantity output. It would be better to be formed in the normalization of prices, as follows:

$$Y = P_c.Cr.(L_c, K_c) + P_m.Mr.(L_m, K_m) \tag{4}$$

where *Pm* and *Pc* are constant unitary units of money prices associated with the output quantity of *Mr*. and *Cr*. From this model can be formed marginal products of both labor (M_L, C_L) and capital (M_K, C_K) which can be made proportionally between sectors, is a derivative of the production function of inputs, namely:

$$\frac{M_L}{C_L} = \frac{M_K}{C_K} = 1 + \delta \tag{5}$$

The notations in equation (5), C_M show the external effects of military output to the private sector, and δ denote the relative factor productivity difference between the two sectors. If $C_M > 0$ and/or $\delta > 0$, an increase in military output will cause a large growth rate of total Y output, from the sum of *M* and *C*, using the given production function *L* and *K*.
Equation (5) is equivalent to:

$$\frac{P_m.Mr_L}{P_c.Cr_L} = \frac{P_m.Mr_K}{P_c.Cr_K} = 1 + \delta \tag{6}$$

Equation (6) shows the comparison between the productivity of marginal factors between different production depending on the prices used on sectoral outputs.

Differentiation of (3), (1), and (2) forms the econometric speech of the growth equation, as follows:

$$\dot{Y} = \frac{C_L.L}{Y}\dot{L} + C_K.\frac{I}{Y} + \left(\frac{\delta}{1+\delta} + C_M\right)\frac{M}{Y}\dot{M} \tag{7}$$

where the dot notation indicates the rate of change in proportion or growth rate. *I/Y* and *M/Y* are notations of the ratio of investment and military spending to total output. While *I = dK* which is a net investment.

Using equation (1) and the constant elasticity of *C* to *M* equation (5) can be changed in the form of:

$$\dot{Y} = \alpha\left(\frac{I}{Y}\right) + \beta(\dot{L}) + \left(\frac{\delta}{1+\delta} - \theta\right)\frac{M}{Y}\dot{M} + \theta.\dot{M} \tag{8}$$

Where :

$$\dot{Y} = \left[\frac{dY}{Y}\right], \quad \dot{L} = \left[\frac{dL}{L}\right], \quad \dot{M} = \left[\frac{dM}{M}\right], \quad \theta = C_M\left[\frac{M}{Y-M}\right], \quad \alpha = C_K, \quad \beta = \left[\frac{C_L.L}{Y}\right] \tag{9}$$

The difference in (7) with (8) is that in equation (7) hypothesis testing is possible only if the variables C_M and δ are zero. However this would cause the coefficient of *(M/Y)* to be zero thus reducing the estimate of the standard growth equation. Using equation (8), we can be estimated

separately to identify separately from the "externality effect" C_M and the "marginal factor productivity differential effect" δ.

3. Result and Discussions

Much debate has emerged against the defense-growth model that begins with the Feder-Ram model. Some experts provide support, such as Deger and Sen that characterize the externality model of Feder-Biswas-Ram as "a splendid empirical workhorse to investigate the impact of military expenditure on growth."[17] Mintz and Stevenson (1995) seen using formal justification to include military expenditure as explanatory variables in growth regression analysis with single-equation growth analysis, based on neoclassical growth theory as its foundation.[18] Or, Biswas and Ram state, at least stands out well on the framework of neoclassical production functions.[4] His famous approach is the emergence of a direct link from the theoretical model to the econometric specification.

However, many critics say that Ram's study does not include other independent variables that also affect economic growth. From the Keynesians see that neoclassical approach is not able to answer the problem in the short term, especially if there are shocks encountered. Full economic conditions are only limited to long-term models that are difficult to implement in reality.

But Dunne, Smith, and Willenbockel said formation of the model faced many inaccuracies. Feder-Ram model that notation of differences in factor of marginal productivity between sectors, as in the following:

$$\frac{M_L}{C_L} = \frac{M_K}{C_K} = 1 + \delta \tag{10}$$

actually gives some wrong interpretations. In empirical literature, non-zero is generally interpreted to reflect a situation in which one sector is more efficient or less productive in its factor users than others.[19]

For example, Ward et al estimates a negative sign for Taiwan which can be inferred "that in comparison to the civil sector, the military sector is realized more inefficiently".[20] Antonakis[15] and Atesoglu and Mueller[5] write something similar: "Without a strong competitive pressure that leads to ... efficiency in the management and use of resources, it can be argued that marginal productivity factors are much lower in the defense sector". Such interpretations do not conform to a recognized model of theory.

Another argument that the approach to capture some of the behavior of off-the-production functions is not appropriate. The derivation of the empirical growth equation:

$$\dot{Y} = \frac{C_L.L}{Y}\dot{L} + C_K.\frac{I}{Y} + \left(\frac{\delta}{1+\delta} - \theta\right)\frac{M}{Y}\dot{M} + \theta\dot{M} \tag{11}$$

is specific to a certain level which is not different for intra-sectoral organizations. The model was built incapable to calculate the intra-oriental organizational inefficiency. The next question is whether the migration of a resource that raises real GDP is what the social really wants can not be answered without knowing where the relative *P* price is used in an adequate *Y* calculation that reflects the social marginal rate of substitution, the exchange of *M* for *C*. If that is the case, the non-zero δ reflects the situation of the product mix in a large economy and the allocation of inter-sectoral factors in eeconomy as a whole is inefficient. There is not much we can do to convert inputs into outputs in individual sectors. In theoretically, there are many econometric problems in estimating the Feder-Ram model. In the Feder-Ram equation where econometrically it can be derived as:

$$\dot{Y} = \beta_1\dot{L} + \beta_2\left(\frac{I}{Y}\right) + \beta_3\frac{M}{Y}\dot{M} + \beta_4.\dot{M} + \varepsilon \tag{12}$$

It can not be indicated which is a variable and which serves as a parameter. This equation treats capital (as a form of influence) is asymmetric, and it is clear that $C_L.L/Y$ chills as a constant β_1, $C_K.I/Y$ compilation is divided as a parameter and a variable, $\beta_2.I/Y$. It is not clear from where error origin and why it is treated as white noise.

We see the example, Wijeweera & Webb [21] examined the economic growth for Sri Lanka using the Feder-Ram usage model [22], Atesoglu Model [23] and Halicioglu [24]. In the Feder-Ram model the results showed t-statistics for each major coefficient (*L* and *K*) were positive, but not statistically significant in the experiments included for military-controlled operations (*M*). Since wages are insignificant it can not be censored whether or not pngelpangin positively affects economic growth in Sri Lanka. Not only are the coefficients sttistically insignificant, but R^2 is also very low which means that the model is not strong enough to get out of the economy. All this leaves doubt on the Feder-Ram model to see the relationship between space and economic growth in the case of Sri Lanka.[21]

4. Conclusion

The Feder-Ram model of economic growth is very broad by people where its limitations are tried to be overcome by various circles of economist.

Dunne, Smith, and Willenbockel criticize by stating that there is no solid basis and econometric reason for using this Feder-Ram model. As for the various simultaneous problems by creating a growth rate of the same dimensions, when the combination of the military is constant, the net output will determine the movement of the military. The multicolinearity between the two final forms yields a large standard error and the estimated inaccuracy (not appropriate forecast) of the parameters of the externalities. The model is static, there is no lagged regressors or dependent variables, which pose a major problem in time-series in cross-section.[19]

The next model of Feder-Ram was developed by Mintz & Huang with some modifications. Mintz & Huang, argued that the externality effect of the military and non-military sectors is different. Therefore, the effects of externalities of the military public sector, the non-military public sector, and the private sector are included in the production function separately.[25]

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