Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 02, No. 03, April-May 2022
http://journal.hmjournals.com/index.php/JECNAM

Applied Mathematics Model: The Effect of National Defense on Economic Growth seen from the Supply Side

Posma Sariguna Johnson Kennedy

Universitas Pertahanan/Universitas Kristen Indonesia

Email: posmahutasoit@gmail.com

***Abstract**: The study aims to develop a mathematical model that can explain the effect of national defense on economic growth from various works of literature and developments. The overall impact of military spending on growth can be analyzed by dividing the economy into sectors. We wish to demonstrate the formation of a general mathematical model. The model building is based on the neoclassical production function approach developed by Feder and Deninson. The source of Deninson's growth model uses a supply-side description of changes in aggregate output. It describes aggregate growth in terms of changes in capital and labor. The resulting aggregate growth equation shows the combination of externality and productivity effects.*

Keywords: Applied Mathematical Model, Economic Growth, National Defense, Supply Side.

1. INTRODUCTION:

According to Benoit[1][2], defense spending will stimulate economic growth, not suppress it. For example, it contributed to the civil economy indirectly through the provision of education and training that can enhance human capital capabilities. Military power can also stimulate research and development and production activities that spread to the civilian sector or ordinary people.

In contrast to Benoit, Joerding[3] argues that economic growth has an impact on government spending. Joerding stated that developing countries would strengthen themselves to face foreign or domestic threats by increasing their military spending. Since Benoit[1] researched the relationship between military spending and economic growth, there has not been a dominant argument explaining the effect of defense spending on economic growth. Until now, there is still a debate about the various arguments of experts [4].

One of the approaches commonly used in research on the relationship between defense spending and economic growth is "*the neoclassical production function approach". *Heo & DeRouen[5] added, "That is by reviewing *the supply-side description* through changes in *aggregate output*". Sandler & Harley[13] stated, "This theory is widely used because it is built from a consistent theoretical structure". It has also been described by Ram[6], Biswas &

Ram[7], Atesoglu & Mueller[8], Mintz & Huang[9][10], Ward & Davis[11], Macnair et al.[12].

This theory assumes that, "Real output per capita and capital stock growth will be constant in a certain period even though there are short-term fluctuations. It is also assumed that an increase in labor and capital input at a steady level will also increase its aggregate output at a steady level. Then changes in aggregate output will be explained by changes in capital and labor".[14]

Productivity also contributes to economic growth and grows towards a *steady rate* in the *long run*. "The production function approach will explain economic growth through changes in *capital*, *labor*, and productivity. Technology is explained by the *output* generated from the amount of *labor* and *capital* used in production"[15]. Denison[16] also explains that advanced technology will give way to production at lower costs.

Since technological developments significantly contribute to economic development, it is very important to study the effect of defense spending on economic growth by including technological developments in the *defense-growth model*. The model's technological developments are meaningful considering that technology has been reflected as part of economic and social integration [17].

Based on the explanation above, this study aims to develop a mathematical model that can explain the influence of national defense on economic growth from various literature and developments. Developed from one sector to four sector models so that can make a general mathematical model.

2. RESEARCH ELABORATIONS:

We can analyze the overall effect of military spending on growth by dividing the economy into sectors. These sectors form their externalities that affect other sectors. The model formed is based on the neoclassical production function approach developed by Feder[18] and Denison[16].

"Denison's *source-of-growth* model uses a *supply-side description* of changes in *aggregate output*, which explains *aggregate growth* in terms of changes in *capital* and *labor*"[16]. Feder[18] writes that, "Aggregate growth is related to changes in *capital* and *labor* through the *underlying production function*. He built *a two-sector production function model* consisting of the *export* and *non-export* sectors".

Adapting this, Ram[6] also developed a two-sector model consisting of the *government* and *private sectors*. "This model states that the *output* in each sector depends on the *input* from *labor* and *capital*. In this model, the military sector is part of the government sector. The approach implicitly assumes that the military sector is the same as *the rest of the government sector*"[5]. Some critics argue this opinion, the study of Ram does not include other independent variables that also affect economic growth.

Mintz & Huang[9][10] tried to overcome the limitations of the Ram model by adding it to three economic sectors, namely the *government, military, and civilian sectors*. It is realized that all effects of the military sector affect growth and can also come from the external effects of military spending.

This Ram model was also refined by Mueller & Astelogu[19], who analyzed *growth* through technological changes in their research, " *the economic effect of defense spending on growth*. The *defense-growth model* by dividing the economy by sector have been used in various kinds of literature (see Biswas and Ram[7], Alexander[20], Atesoglu & Mueller[8], Mintz & Huang[9][10], Ward & Davis[11], Heo & DeRouen[5], Heo[21], DeRouen & Heo[22], Heo & Eger[23].

Antonakis[4] suggested dividing the economy into four sectors in building a *growth* model for its development. "This model aims to capture the effect of military spending on growth without ignoring the influence of other influential variables"[4]. These variables are arranged in the framework:

1) The economy comprises four sectors, which are *mutually exclusive* and *exhaustive* based on *output*. These sectors are called: the military sector, the non-military sector, the export sector, and *the rest of the economy*, which have important interactions with *output*.
2) *Labor and capital* are provided as inputs for all sectors. Some sectors can affect the *output* of other sectors either negatively or positively. This effect is called an externality because it is not reflected in market prices.
3) Each sector has a different influence on externalities.

Heo & Eger[23] also developed a four-sector model such as Antonakis. The four sectors are military, non-military government, exports, and the rest of the economy. Heo & Eger builds a model based on Mintz & Huang's[9][10] research that looks at sectors in the economy and their externalities and Mueller & Atesoglu's[19] research that looks at *technological change*.

There are two valid reasons for combining these two approaches [5][24]:

1) First, "According to Mintz & Stevenson[25] the *defense sector* must be separated from the *non-defense sector* because the defense industries have different incentives. The implication is that the productivity of the military sector is affected by very different inputs from that of the *private sector*. For this reason, it is assumed that the economy consists of three sectors, namely: the military sector (*M*), the non-military government sector (*N*), and the private sector (*P*) [9][10]. Following Feder[18], the *output* of these sectors is assumed to depend on the inputs of *labor* (*L*) and *capital* (*K*). Following Huang & Mintz[26], it also assumes that the two components of the government sector have separate externality effects on *private*-sector *output*. The effects of externalities may benefit the *private sector* by generating *technological spillovers*. On the other hand, negative externalities can also suppress economic growth, for example, government regulations. According to Cornes & Sandler[27], when government activities affect the production capacity of the private sector in the absence of price and market competition, it has an externality effect on the output of the *private sector*".[5]
2) Second, it emerges from Mueller & Atesoglu's[19] opinion, incorporating technological change into the model. This technological change may vary each year, which is assumed to be a certain average level over the long term as $e^{\lambda t}$. There are two benefits to including technology growth at this level:
   - Chan[28] explains that the *defense-growth relationship* is not linear. By including *technological progress* with the level of $^{\lambda t}$. "It is possible to examine the relationship between *defense expenditure* and *economic growth* in a non-linear context"[5].
   - Incorporating *technological change* into the model can divide "*defense spending* into two components: (1) *defense growth effect* and (2) *defense share to GDP* from the *defense size effect sector*. According to Mueller & Atesoglu[19], this separation is particularly significant because changes in the *rate of defense spending* have an immediate effect on economic growth when the *size effect* spreads over more than one year according to the *size of the defense sector* and the size of *the size. Of the whole economy*"[5].

Furthermore, Heo & Eger[23] conducted research on the relationship between state defense spending and economic growth by including the level of technological development. Heo also explained that economic *output comes from labor and capital production function*s. Meanwhile, the economy consists of four sectors, namely the military, non-military, exports, and *the rest of the economy*. Heo divides into four sectors for several reasons [23]:

1) Ward & Davis[11] show that, “The *government sector* and the *private sector* have different products, whereas the government usually has lower productivity. Therefore, the government sector needs to be separated from the *private sector*.” See also Ram[6].
2) Alexander[20] mentions that, “The military sector has a different position than *the rest of the economy*. So it must be separated between the military sector and the non-military sector. The military sector has its incentives and therefore has a different impact on *growth* than other sectors of the economy.”
3) Many studies show that there is a positive influence of exports on growth so that the export sector is also separated. See also Feder[18].
4) Alexander[20] argues that, “To capture the effect of *military spending* on *growth, you must use* at least four sectors if you want to ignore other macroeconomic variables”.
5) Feder[18] also argues, “A substantial effect between *marginal productivity factors* in export-oriented and non-export oriented industries. Export-oriented industries usually have higher productivity”.
6) Alexander[20] states that, “Assuming four sectors in the economy can include all *indirect channels* such as *investment/capital, labor/employment*, and exports in the *growth equation*”.

3. RESULTS AND DISCUSSION:

A. Development of Modeling the Effect of National Defense on Economic Growth

The theory of economic growth (*Y*) that develops is to divide it into more sectors in the economy, with the development [29]“:

1) One Sector Growth Theory

One of the neoclassical growth theories was introduced by Solow[15], where there is no *channel* from *government spending* that affects *long-run economic growth*. Thus there is no government spending sector in it. The assumption formed by Barro & Martin[30] is a *closed economy*, *one-sector production technology*, where *output* is homogeneous goods that can be consumed and invested. Investment generates new physical capital, and it depreciates at a constant rate. *Households* and *firms* are considered joint units (which own *inputs* and manage the technology that transforms *inputs* into *outputs*), the market being ignored first. The *output current* produced at time *Y(t)* is influenced by the production function of *capital (K)* and *labor (L)* which depends on time (*t*) to reflect the effects of technological development, which is described as follows:

$$Y(t) = F[K(t), L(t), t] \quad (1)$$

2) Two-Sector Growth Theory

Deninson's *source-of-growth* model uses a *supply-side description* of changes in *aggregate output*, which explains *aggregate growth* in terms of changes in *capital* and *labor*. Feder[18] (1983) writes that aggregate growth is related to changes in *capital* and *labor* through the *underlying production function*. Feder built *a two-sector production function model* consisting of the *export sector (E)* and the *non-export sector (R)*, namely:

$$Y = E + R \tag{2}$$

Inspired by this model, Ram[6] also developed a two-sector model, which consists of the *government sector (G)* and the *private sector (P)*. This model states that the *output* in each sector depends on the *input* from *labor* and *capital*. In this model, the military sector is part of the government sector. The approach implicitly assumes that the military sector is the same as *the rest of the government sector*. Two-sector model from Ram:

$$Y = G + P$$

" (3)

3) Three-Sector Growth Model

Mintz & Huang[9][10] tried to overcome the limitations of the Ram's[6] model by adding it to three economic sectors, namely the *government sector (N)*, the *military sector (M)*, and the *civilian sector (P)*. It is realized that all effects of the military sector affect growth and can also come from the external effects of military spending. The Mintz & Huang models are:

$$Y = N + M + P \tag{4}$$

Mueller & Astelogu[19] analyzed *growth* through technological changes. Mintz & Huang[9][10] argue that the externality effect of the government spending sector on the military and non-military is different because it has different production functions. Inspired by the study Heo & DeRouen[5] developed this model, "The production function model becomes:

Production Function

Formally, the production function model of the three economic sectors, namely military *(M)*, non-military *(N)*, and *private (P)*. We added security as an additional production factor which is the responsibility of defense in dealing with threats. The model is given by the following aggregate production function:

$$\begin{aligned} M &= A(t)F(L_m, K_m, S) \\ N &= B(t)G(L_n, K_n) \\ P &= C(t)H(L_p, K_p, M, N) \end{aligned} \tag{5}$$

Technological Development Level (Φ_i)

The level of technological development between sectors based on the P (*private*) sector is written as follows:

$$\begin{aligned} A(t)/C(t) &= 1 + \Phi_m \\ B(t)/C(t) &= 1 + \Phi_n \end{aligned} \tag{6}$$

Marginal Productivity (δ_i)

The marginal productivity of *labor* and *capital* can be written based on the *private* sector (P) as follows:

$$F_l / H_l = F_k / H_k = 1 + \delta_m$$
$$G_l / H_l = G_k / H_k = 1 + \delta_n \qquad (7)$$

Economic Input
Total total *input* is the sum of labor and capital:

$$L = L_m + L_n + L_p$$
$$K = K_m + K_n + K_p \qquad (8)$$

Total Output
Total economic output, GDP *(Y)*, is the sum of *the outputs* of all sectors, as an objective function, namely:

$$Y = M + N + P \qquad (9)$$

The economy grows over time, the above equation can be done by *differentiation with respect to time* (for example, *X'(t)=dX(t)/dt* which is then abbreviated to *dX*) of each *equation*, namely:

$$M = A(t)F(L_m, K_m, S)$$
then: $M' = dM = F.dA + AF_{l.}\, dL_m + AF_{k.}\, dK_m + AF_s\, dS$
$$N = B(t)G(L_n, K_n)$$
then: $N' = dN = G.dB + BG_{l.}\, dL_n + BG_{k.}\, dK_n$
$$P = C(t)H(L_p, K_p, M, N)$$
then : $P' = dP = HdC + CH_l dL_p + CH_k dK_p + CH_m dM + CH_n dN$

" (10)

The total differential summing all the *outputs* gives the result:

$$dY = FdA + AF_l dL_m + AF_k dK_m + AF_s dS + GdB + BG_l dL_n + BG_k dK_n + HdC + CH_l dL_p + CH_k dK_p + CH_m dM + CH_n dN \qquad (11)$$

4) Four Sector Growth Model

Heo & Eger[23] further divides economic *output* into four sectors, *the four-sector production function model of the economy*. The four sectors are *military* (*M*), non-*military government* (*N*), *exports* (*E*), and *the rest of the economy* (*R*). "The Heo & Eger models are:

$$Y = M + N + E + R \qquad (12)$$

The model is built based on the four-sector model.

By the assumptions built, formally, the production function model of the four economic sectors. We added security as an additional production factor which is the responsibility of defense in dealing with threats. The model is given by the following aggregate production function, namely:

Military sector : $M = A(t)F(L_m, K_m, S)$
Non-military sector : $N = B(t)G(L_n, K_n)$
Export sector : $E = C(t)H(L_e, K_e, M, N)$
Rest of economy : $R = D(t)I(L_r, K_r, M, N, E)$

(13)

The rate of technological progress differs between sectors, *A(t), B(t), C(t), and D(t),* so they are entered into the model separately. However, Heo & Eger[23] assume that technological change between sectors will differ proportionally from each other. Thus the development of technology between sectors can be written as follows:

$$A(t)/D(t) = 1 + \Phi_m$$
$$B(t)/D(t) = 1 + \Phi_n$$
$$C(t)/D(t) = 1 + \Phi_e$$

(14)

The proportion factor of technological development Φ_i ($i=m,n,e$) is constant and unknown.

By Hicks, the *form of neutral technical change* as *constant factor shares*. The point is that changes in technology will not change *the share of income going to the factors of production and the factor ratios.*[19]

Productivity factors may differ between sectors. The *marginal productivity of labor and capital* also differs between sectors. The marginal productivity of a factor of production is a change or increase in *output* formed by adding one unit of a factor of production by using several other factors of production. Assuming the marginal productivity of a factor of production from a sector is $1+\delta_i$ ($i=m,n,e$). The marginal productivity of *labor* and *capital* can be written based on sector *R* as follows:

$$F_l / I_l = F_k / I_k = 1 + \delta_m$$
$$G_l / I_l = G_k / I_k = 1 + \delta_n$$
$$H_l / I_l = H_k / I_k = 1 + \delta_e$$

(15)

Where F_i, G_i, H_i, I_i, ($i = l, k$) are the *marginal products of labor and capital* in the four sectors, which are partial derivatives of the production function with respect to the *input.* It is assumed that the *marginal products of labor and capital* in sector *i* can be higher or lower than the factor $1+\delta_i$. The *factor productivity differential* δ_i is a constant and unknown, which can be any value, including zero.

Total economic output *(Y)* is the sum of the various *outputs*. While the total total inputs is the sum of labor and capital:

$$L = L_m + L_n + L_e + L_r$$
$$K = K_m + K_n + K_e + K_r$$

(16)

Lowercase letters on inputs identify the allocation of inputs from sectors. *K* and *L* define the total *inputs* supplied at a point in time. *K* is the *total capital,* and *L* is the *total labor* in the economy.

The total *output,* which is an objective function, is the sum of *the outputs* of the four sectors, namely:

$$Y = M + N + E + R$$

" (17)

Using all the equations and doing a mathematical derivation can be formed an equation for estimation is as follows:

$$\left(\frac{dY}{Y}\right) = \lambda + e^{\lambda l}\psi_l\left(\frac{dL}{L}\right) + e^{\lambda t}\psi_k\left(\frac{dK}{K}\right) + e^{\lambda t}\psi_s\left(\frac{dS}{S}\right) + [\pi_m(M/Y) + e^{\lambda l}\psi_m]\left(\frac{dM}{M}\right)$$
$$+ [\pi_n(N/Y) + e^{\lambda t}\psi_n]\left(\frac{dN}{N}\right) + [\pi_e(E/Y) + e^{\lambda t}\psi_e]\left(\frac{dE}{E}\right)$$
$$+ \lambda\pi_m\left(\frac{M}{Y}\right) + \lambda\pi_n\left(\frac{N}{Y}\right) + \lambda\pi_e\left(\frac{E}{Y}\right)$$

(18)

The growth equation model that is formed can capture the direct effect of each sector *i* on economic *growth*. Parameter ψ_i describes the effect of externalities from sector *i*. The growth of sector *i* has an externality effect on *the rest of the economy*. Thus, these sectors are expected to provide the creation of externalities and have different productivity.

B. Generalization of the Economic Growth – National Defense Model

The Feder[18]-Ram[6] model is widely used by economists and developed by various groups. Namely: development of the three-sector *growth-defense model* of Mintz-Huang[9][10], the inspiration of Murdoch, Ron Pi, & Sandler[31], the economic sectors and their externalities, technological change by Mueller & Atesoglu[19] (1993), a three-sector *growth-defense model* with technology from Heo & DeRouen[5], and a four-sector *growth-defense model* (Antonakis[4], Heo & Eger[23]) that create a non-linear effect on spending military and other sectors. With so many developments by economists, the defense-growth model initiated by the Feder-Ram model needs to be generalized.

Inspired by Cuaresma & Wörz's[32] research framework, a general model can be made, "The total production of the economy *(Y(t))* consists of the production of government spending *(X(t))* and the rest of the economy *(N(t)).* It is assumed that the production of government spending consists of several sectors (e.g., the military sector and the non-military sector), so that:

$$X(t) = \sum_{i=1}^{S} X_i(t)$$

(19)

Suppose the production of all sectors in the economy affects *the rest of the economy* and gives an externality effect, then:

$$N(t) = F(K_N(t), L_N(t), X_1(t), X_2(t), \ldots, X_S(t))$$

(20)

Where $K_N(t)$ and $L_N(t)$ *are* the stock of capital and labor used in *the rest of the economy*. For example, the production of government spending in sector *i* is given by:

$$X_i(t) = G_i(K_i(t), L_i(t)) \qquad i = 1, \ldots, S$$

(21)

Where $K_i(t)$ and $L_i(t)$ are the stock of capital and labor used by sector *i*."

It is further assumed that the productivity factor of each sector is different or specific to the specification factor *i>-1*, then:

$$\frac{\partial G_i / \partial K_i}{\partial F / \partial K_N} = \frac{\partial G_i / \partial L_i}{\partial F / \partial L_N} = 1 + \delta_i \qquad i = 1, \ldots, S \tag{22}$$

Using the fact that:

$$\frac{dN}{dt} = \frac{\partial F}{\partial K_N}\frac{dK_N}{dt} + \frac{\partial F}{\partial L_N}\frac{dL_N}{dt} + \sum_{i=1}^{S}\frac{\partial F}{\partial X_i}\frac{dX_i}{dt} \tag{23}$$

and identity

$$Y = N + \sum_{i=1}^{S} X_i \tag{24}$$

We use some manipulation, and it can be written as follows:

$$\frac{dY/dt}{Y} = \frac{\partial F}{\partial K_N}.\frac{dK/dt}{Y} + \frac{\partial F}{\partial L_N}.\frac{dL/dt}{Y} + \sum_{i=1}^{S}\left(\frac{\partial F}{\partial X_i} + \frac{\delta_i}{1+\delta_i}\right).\frac{dX_i/dt}{X_i}.\frac{X_i}{Y} \tag{25}$$

Where :

$$K = K_N + \sum_{i=1}^{S} K_i \quad \text{and} \qquad L = L_N + \sum_{i=1}^{S} L_i \tag{26}$$

Following Feder[18], it is assumed that there is a linear relationship between marginal labor productivity and average output per worker, then by:

$$\frac{\partial F}{\partial L_N} = \gamma\left(\frac{Y}{L}\right) \tag{27}$$

We can write the equation as:

$$\frac{dY/dt}{Y} = \beta.\frac{dK/dt}{Y} + \gamma.\frac{dL/dt}{Y} + \sum_{i=1}^{S}\left(\frac{\partial F}{\partial X_i} + \frac{\delta_i}{1+\delta_i}\right).\frac{dX_i/dt}{X_i}.\frac{X_i}{Y} \tag{28}$$

where the *marginal* productivity of capital in *the rest of the economy* is a constant assumption.

Although it can analyze the above equation empirically to determine whether each sector has a different effect on growth, it cannot empirically identify the effect of externalities ($\partial F / \partial X_i$) and differences in productivity ($\delta_i / (1+\delta_i)$).

If the production function of *the rest of the economy is* as:

$$N(t) = F(K_N, L_N, X_1, X_2, \ldots, X_S) = \left(\prod_{i=1}^{S} X_i^{\psi i}\right)\tilde{F}(K_N, L_N) \tag{29}$$

With a parameter $\psi_i \in \Re, i = 1, \ldots S.$, then parameter implication :

Journal of Electronics, Computer Networking and Applied Mathematics
ISSN: 2799-1156
Vol: 02, No. 03, April-May 2022
http://journal.hmjournals.com/index.php/JECNAM

$$\frac{\partial F}{\partial X_i} = \psi_i \frac{N}{X_i} \tag{30}$$

So can write the equation as:

$$\frac{dY/dt}{Y} = \beta.\frac{dK/dt}{Y} + \gamma.\frac{dL/dt}{Y} + \sum_{i=1}^{S}\left[\frac{\psi_i.dX_i/dt}{X_i}.\left(1 - \frac{\sum_{i=1}^{s} X_i}{Y}\right) + \frac{\delta_i}{1+\delta_i}\frac{dX_i/dt}{X_i}.\frac{X_i}{Y}\right] \tag{31}$$

able to estimate ψ_i and δ_i for $i=1,\dots, S$ empirically.

Based on the analytical framework above, without making assumptions on the shape of the external effect function of each sector, then from the previous equation:

$$\frac{dY/dt}{Y} = \beta.\frac{dK/dt}{Y} + \gamma.\frac{dL/dt}{Y} + \sum_{i=1}^{S}\left(\frac{\partial F}{\partial X_i} + \frac{\delta_i}{1+\delta_i}\right).\frac{dX_i/dt}{X_i}.\frac{X_i}{Y} \tag{32}$$

If it supposes there are k sectors other than *the rest of the economy*, then we can write the equation can as:

$$\frac{\Delta Y_{it}}{Y_{it}} = \alpha + \beta.\frac{\Delta K_{it}}{Y_{it}} + \gamma.\frac{\Delta L_{it}}{Y_{it}} + \sum_{k=1}^{S} \phi_k \frac{\Delta X_{k,it}}{X_{k,it}}\frac{X_{k,it}}{Y_{it}} + \varepsilon_{it} \tag{33}$$

Based on the specification that high technology may from a sector provide higher productivity, we can analyze externalities empirically from differences in productivity of each sector. With the specific equation as in the previous:

$$\frac{dY/dt}{Y} = \beta.\frac{dK/dt}{Y} + \gamma.\frac{dL/dt}{Y} + \sum_{i=1}^{S}\left[\frac{\psi_i.dX_i/dt}{X_i}.\left(1 - \frac{\sum_{i=1}^{s} X_i}{Y}\right) + \frac{\delta_i}{1+\delta_i}\frac{dX_i/dt}{X_i}.\frac{X_i}{Y}\right] \tag{34}$$

It can be written as follows (if, for example, there are three *k sectors* other than *the rest of the economy*):

$$\frac{\Delta Y_{it}}{Y_{it}} = \alpha + \beta.\frac{\Delta K_{it}}{Y_{it}} + \gamma.\frac{\Delta L_{it}}{Y_{it}} + \sum_{k=1}^{3} \rho_k \frac{\Delta X_{k,it}}{X_{k,it}}\left(1 - \frac{\sum_k X_{k,it}}{Y_{it}}\right) + \sum_{k=1}^{3}\pi_k \frac{\Delta X_{k,it}}{X_{k,it}}\frac{X_{k,it}}{Y_{it}} + \varepsilon_{it} \tag{35}$$

with: $$\pi_k = \frac{\delta_k}{1+\delta_k} \tag{36}$$

Where :

$\frac{\Delta Y_{it}}{Y_{it}}$ is the period average of the annual growth of real GDP for country *i*

$\frac{\Delta K_{it}}{Y_{it}}$ is a proxy for the period's average share of investment in GDP

$\frac{\Delta L_{it}}{Y_{it}}$ can be proxied by using the average period of population growth.

4. CONCLUSIONS :

We want to create a economy growth-national defense model to see the influence of the defence sector on the economy and externalities between sectors. According to the Feder-Ram equation, a general model is formed in this study. Equations are built from several sectors. The mathematical equation of the growth model shows the results in general, namely:

$$\frac{\Delta Y_{it}}{Y_{it}} = \alpha + \beta.\frac{\Delta K_{it}}{Y_{it}} + \gamma.\frac{\Delta L_{it}}{Y_{it}} + \sum_{k=1}^{3} \rho_k \frac{\Delta X_{k,it}}{X_{k,it}} \left(1 - \frac{\sum_k X_{k,it}}{Y_{it}} \right) + \sum_{k=1}^{3} \pi_k \frac{\Delta X_{k,it}}{X_{k,it}} \frac{X_{k,it}}{Y_{it}} + \varepsilon_{it} \quad (37)$$

Thus the *aggregate growth equation* is viewed as the result of a combination of the effects of externalities and productivity effects, namely:

- The growth rate interaction of a particular sector, which includes the combination of the effects of technological progress and differential productivity of the sector, and
- Externalities between certain sectors and *the rest of the economy* are part of the share of the sector's output growth rate.

5. REFERENCES :

[1] Benoit, E. (1973). *Defence and Economic Growth in Developing Countries*. Lexington.

[2] Benoit, E. (1978). “Growth and Defence in Developing Countries”, *Economic Development and Cultural Change*. Vol.26 (2) : 271-280.

[3] Joerding, W. (1986). “Economic Growth and Defence Spending”, *Journal of Development Economics*, Vol. 21.

[4] Antonakis, Nicholas. (1999) “Guns versus Butter: A Multi-Sectoral Approach to Military Expenditures and Growth with Evidence from Greece, 1960-1993”, *Journal of Conflict Resolution*, Vol. 43 (4) : 501-520.

[5] Heo, Uk. and Karl DeRouen Jr. (1998) “Military Expenditures, Technological Change, and Economic Growth in The East Asian NICs”, *Journal of Politics*, Vol. 60: 830-46.

[6] Ram, Rati. (1986) “Government Size and Economic Growth: A New Framework and some Evidence from Cross-Section and Time-Series Data”, *American Economic Review*, Vol.76 (1):191-203.

[7] Biswas, B. and Ram, R. (1986). “Military Expenditures and Economic Growth in Less Developed Countries: An Augmented Model and Further Evidence”, *Economic Development and Cultural Change*, Vol. 34.

[8] Atesoglu, H. Sonmez & Mueller, J. Michael. (1990). “Defence Spending and Economic Growth”, *Defence Economics*, Vol. 2(1) : 19-27.

[9] Mintz, Alex and Huang, Chi. (1990) “Defence Expenditures, Economic Growth and the Peace Dividend”, *American Political Science Review*, Vol. 84 (4) : 1283-1293.

[10] Mintz, Alex and Huang, Chi. (1991) “Guns versus Butter: The Indirect Link”. *American Journal of Political Science*, Vol. 35 (3) : 738-57.

[11] Ward, M.D. and Davis, D.R (1992) “Sizing up the Peace Dividend: Economic growth and military spending in the US, 1948-1996”, *American Political Science Review*, Vol. 86 (3).

[12] MacNair, J.C.; Murdoch, J.C & Pi C-R. (1995). “Growth and Defense: Pooled Estimates for the NATO Alliance, 1951-1988”. *Southern Economic Journal*. Vol. 61 (3):846-860.

[13] Sandler, Todd and Hertley, Keith. (1995). *The Economics of Defence. Cambridge surveys of Economic Literature*, UK: Cambridge University Press.

[14] Solow, Robert M. (1970). *Growth Theory: An Exposition*. New York: Oxford University Press.
[15] Solow, Robert M. (1957). "A Technical Change and the Aggregate Production Function", *Review of Economics and Statistics*, Vol.39 (3), pp. 312-320.
[16] Denison, Edward F. (1985). *Trends in American Economic Growth*, 1929-1982. Washington, DC: Brookings Institution.
[17] Hofheinz, Ray and Calder. (1982). *The East Asia Edge*. New York: Basic Book.
[18] Feder, Gershon. (1983) "On Exports and Economic Growth", *Journal of Development Economics*, Vol. 12: 59-73.
[19] Muller, M. and Atesoglu, H. (1993) "Defence Spending Technological Change and Economic Growth in the United States", *Defence Economics*, Vol. 4 (3).
[20] Alexander, W. Robert J. (1990) "The Impact of Defence Spending on Economic Growth: A Multi-Sectoral Approach to Defence Spending and Economic Growth with Evidence from Developed Economies", *Defence Economics*, Vol.2 : 39-55.
[21] Heo, Uk. (1999). "Defense Spending and Economic Growth in South Korea: The Indirect Link", *Journal of Peace Research*, Vol. 36 (6): 699-708.
[22] DeRouen Jr, Karl and Heo, Uk. (2001). "Modernisation and the Military in Latin America", *British Journal of Political Science*, Vol.31 (3) : 475-496.
[23] Heo, Uk. and Eger, Robert J. (2005). "Paying for Security-Prosperity Dilemma in the United States", *The Journal of Conflict Resolution*, Vol. 49(5): 792-817.
[24] Heo, Uk. (1998) "Modeling The Defense-Growth Relationship Around The Globe". *Journal of Conflict Resolution*, Vol. 42: 637-57.
[25] Mintz, Alex and Stevenson, R. (1995) "Defence Expenditures, Economic Growth and the Peace Dividend: A Longitudinal Analysis of 103 Countries", *The Journal of Conflict Resolution*, Vol.39(2): 283-305.
[26] Huang, C. and Mintz, A. (1991) "Defence Expenditures and Economic Growth: The Externality Effect", *Defence Economics*, Vol. 3(11).
[27] Cornes, Richard and Todd Sandler. (1986). *The Theory of Externalities, Public Goods, and Club Goods*. Cambridge: Cambridge University Press.
[28] Chan, Steve. (1987). *Military Expenditures and Economic Performance. In World Military Expenditures and Arms Transfers*, edited by the U.S. Arms Control and Disarmament Agency. Washington, DC: Government Printing Office.
[29] Kennedy P.S.J., Tobing S.J.L, Toruan R.L., Tampubolon E., and Heatubun A.B. Mathematical Model: The Long-Term Effects of Defense Expenditure on Economic Growth and the Criticism. *J. Phys.: Conf. Ser.* 1114 012118. 2018.
[30] Barro R.J., and Sala-I-Martin, X. (1995). *Economic Growth*. New York: McGraw-Hill.
[31] Murdoch, James C., Chung-Ron Pi and Todd Sandler. (1997). The Impact of Defense and Non-Defense Public Spending on Growth in Asia and Latin America. *Defence and Peace Economics* 8, 205-24.
[32] Cuaresma, J.C and Worz Julia. (2005) "On Export Composition and Growth", *Review of World Economics*, Vol. 141 (1): 33-49.

